UNIVERSITY OF NORTH BENGAL

B.Sc. Honours 4th Semester Examination, 2021

CC10-MATHEMATICS

METRIC SPACES AND COMPLEX THEORY

Full Marks: 60

ASSIGNMENT

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
All symbols are of usual significance.

GROUP-A

1. Answer the following questions: $2\times5 = 10$

(a) Is the set $\mathbb{N}$ of natural numbers compact in the metric space $(\mathbb{R}, d)$, where d is the usual metric? Justify your answer. 2

(b) Check whether the set $\mathbb{Q}$ of rational numbers is connected or not with respect to usual metric. 2

(c) Show that the function $f(z)=|z|;\ z\in\mathbb{C}$ is nowhere differentiable. 2

(d) Prove that no non-constant polynomial function is bounded. 2

(e) Evaluate $\int_\gamma z^3 dz$, where γ is the line segment from 0 to $1+i$. 2

GROUP-B

Answer the following questions $10\times3 = 30$

2. (a) Every compact subset Y of metric space $(X;\ d)$ is closed and bounded. Does the converse true? If not, give example. 3

(b) Let d denotes the discrete metric on a non-empty set X. Prove that X is connected if and only if it is a singleton set. 3

(c) Let $(X; d_X)$ and $(Y; d_Y)$ be metric spaces and F be a non-empty subset of X. If f and g are continuous function from X into Y such that $f(x)=g(x)$ for every x in F, show that $f(x)=g(x)$ for every $x\in\overline{F}$. 4

3. (a) Show that a metric space X is compact iff every real valued continuous function on X is bounded. 4

(b) Let $f(z)$ be non-zero entire function. Show that there exists an entire function $g(z)$ such that $e^{g(z)} = f(z)$. 3

(c) If $f = u + iv$ is an entire function and $u(z) \leq 0$ for all $z \in \mathbb{C}$, then show that f must be constant. 3

4. (a) Let γ be the positively oriented circle $|z| = 3$. Evaluate $\int_{\gamma} \frac{e^z}{z(z-2)}\, dz$. 5

(b) If $w = f(z)$ is an analytic function of $z = x + iy$ such that $f'(z) \neq 0$, show that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) \log |f'(z)| = 0$. 5

GROUP-C

Answer the following questions 5×2 = 10

5. (a) "A bounded set in a metric space may not be totally bounded." — Justify the statement by giving a suitable example. 5

(b) Show that $\cosh\left(z + \frac{1}{z}\right) = a_0 + \sum_{n=1}^{\infty} a_n \left(z^n + \frac{1}{z^n}\right)$, where 5

$$a_n = \int_0^{2\pi} \cos n\theta \cosh(2\cos\theta)\, d\theta$$

GROUP-D

Answer the following questions 5×2 = 10

6. (a) Let $f(z) = e^z$ and its Taylor series expansion centered at i be given by 5

$$f(z) = \sum_{n=0}^{\infty} a_n (z - i)^n$$

Calculate a_3.

(b) Let $(X; d)$ be a metric space. Show that X is connected if and only if every continuous function $f : X \to \{\pm 1\}$ is a constant function. 5

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